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THE IMPORTANCE OF SET THEORY ELEMENTS AND MATHEMATICAL LOGIC IN PRIMARY EDUCATION

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Abstract

Mathematical reasoning and scientific rigorous thought gives the student the possibility to understand other disciplines and look into problems related to nature, life and society, contributes to building and developing the ability to work in an organized and at a sustained pace, enhancing insight and spirit of investigation. At the object of Mathematics, the set theory and mathematical logic elements bring variety in building the concept of natural number, operations with natural numbers and in solving problems, reanimating the lesson and, therefore, making the path to skills' formation safer and more pleasant. The article is part of our graduation paper and approaches aspects related to the use and integration of set theory and mathematical logic elements in lessons of Mathematics, which will lead to the efficient learning of mathematical notions and problem solving and, by this, to the school progress of primary school students.

Key words*:* set theory, mathematical logic, research, creativity, experiment

1. Introduction

Mathematical thought is manifested through a variety of intellectual activities connected to memory and imagination, namely: judgment, reasoning, understanding, explanation, invention, subtraction, induction, analogy, abstraction, generalization, comparison, concretization, classification, division, solving problem-situations, etc. (I., Neacşu, 1988, p. 30). Mathematical competence represents the ability to develop and apply mathematical thought, with the purpose of solving a series of problems occurring in daily situations. One of the vast domains of Mathematics is represented by calculus, which is done in the form of oral or written calculus.

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Relying on a series of algorithms, written calculus becomes quickly automated and operates according to the techniques of computers.

Oral calculus conceptually defines the mental effort made by the student to solve an exercise that requires the activation of knowledge and experience in finding the solution to new, unlearned situations. Through the formative value of oral calculus, its presence is also imposed in the presence of personal computers.

2. The relevance of set theory and mathematical logic elements in building mathematical language

Mathematical language, the language of the most abstract and general concepts, is first introduced with some difficulties. That is why we should ensure, first of all, the understanding of the respective notions, seizing the essence in a language familiar and accessible to children, making, therefore, a compromise on the part of mathematical language. As the understanding of the respective notions is ensured, its scientific name should also be represented. Therefore, as the students advance in the correct understanding of mathematical knowledge, scientific rigorous language is also introduced.

The particularity of teaching Mathematics to 1^{st}-graders is represented by the inductive and analogical methods. The inductive method implies experimenting on the given situation, performing real actions with objects or concepts. Based on the observations made during these concretizations, the students are gradually guided towards concepts. The analogical method relies on a characteristic of Mathematical thought, namely its logical-analogical relevance. Analogies may occur between notions, ideas, theorems, domains.

Before applying the logical properties, the logical operations (negation, disjunction, conjunction, implication, equivalence), practice is done at the level of actions with objects, concrete operations. Hence, the teaching-learning process of Mathematics in the primary cycle implies, first of all, performing concrete actions, operations with objects, which may be subsequently structured and internalized, turning into abstract logical operations.

2.1. Using set theory and mathematical logic elements in teaching numbering

Children of small school age are at the stage of concrete operations. They learn though intuition and direct manipulation of concrete objects, and mathematical activity reproduces, between certain limits, the physical space in which these grow.

The process of building a natural number may be acquired by the child intuitively, by building sets with as many elements and then adding one unit to it. In this way, the student understands numbering more easily and realizes the fact that the last number refers to the totality of objects and not to one single object.

2.2. Creativity in Mathematics and logical-mathematical games

The aim of all *creative education* is to render possible the affirmation and development of the creative potential of each child from the primary cycle. However, all education appreciates reproductive and conforming thought more than the creative and original one.

The logical-mathematical games conducted with special materials from the Dienes kit, which are especially designed to lead children to an intuitive understanding and having an imitative nature for the start, are based on the premise that logical-mathematical games support the discovery of guided and not spontaneous, independent Mathematics.

Logical-mathematical games fall into the following categories: - free games; - games for building sets; - games for arranging pieces in a painting; - games of differences; - games with circles; - games of building pairs; - games of transformations; - games with equivalent sets.

Using logical-mathematical games in the instructive-educational process makes students learn with pleasure, interested in the respective activity, makes the shy students voluble, active, daring and more confident in their intellectual skills, giving their answers more assertiveness and tenacity (Cerghit, 1980).

2.3. Using set theory and mathematical logic elements in teaching-learning arithmetic operations

The starting point in building the notion of addition is represented by operations with sets of concrete objects (the perceptive stage), followed by operations with representations that tend to generalize (the representation stage) so that, eventually, to perform the leap to the mathematical concept of addition (abstract stage). Introducing the addition operation is done by using the reunion of two disjoint sets.

In the concrete stage, the students build sets with concrete objects. Then, the action of building sets using other objects (balloons, sticks, flowers, crayons etc.) is repeated. In the following stage, the students' action focuses on numbering or the composition of a number from two given components.

Subtraction is introduced by using the operation of difference between a set and one of its subsets (the complement of a subset).

In the first concrete stage, from a set of objects sharing one property, a subset of objects is eliminated and it is mentioned how many objects are left in the set. The mental action of the student is related to numbering or decomposing a number into two components, there being given one of these.

The second, semi-abstract stage is characterized by the use if symbolic representations. During this stage, the graphic sign "–" is introduced, explaining its meaning and highlighting the fact that this is written only between numbers.

In the third abstract stage, in which only numbers are used, the specific terminology is introduced (minuend, subtrahend, difference) and there are highlighted the properties of deducting natural numbers, (the operation is possible only if the minuend is higher or equal to the subtrahend, in case of equality, the difference is zero) and compared with the properties of addition (subtraction is not commutative), also highlighting the fact that, in addition, the result (sum) is higher than any of the numbers which are added (the terms), whereas in subtraction, the result (difference) is smaller than the subtrahend.

Multiplication is taught as repeated addition of equal terms, followed by knowledge and acquisition of the technical elements, of how the factors are arranged in achieving the product, as well as reminding the names of the factors and result of multiplication (product), highlighting the function fulfilled by each factor of the product. Therefore, it is necessary to insist upon building the students' skills in placing the factors according to the rule of placing the terms of 1^{st} degree operations, and later to introduce and use the placing of factors in a row, and the product below the multiplicand, in order to achieve space and energy economy and to prepare the transition to division, where the terms are placed only in a row.

To establish a written calculus procedure, we use knowledge of oral calculus, namely the sequential multiplication of the units of different orders of the multiplicand with the multiplier, summing the results. By performing the calculus in writing, it is highlighted the superiority of this calculus compared to the oral one, given by the fact that the product is obtained directly, without intermediary calculi. Also, there are reminded, mentioned and applied the rules established for the other operations related to performing the oral and the written calculus.

Oral multiplication is done starting with the highest-order units, in our case the order of hundreds, thus obtaining the products corresponding to multiplying each order with the multiplier, which are afterwards summed.

Written multiplication is done starting with the lowest-order units, therefore with simple units, then with the tens and hundreds (from right to left), in an analogous way with addition or subtraction.

Division into equal parts, when we know the number of subsets, but we do not know the numbers of elements of each subset, is more easily understood by students.

Quotative division is based on knowledge of the number of elements from each subset, but we do not know the number of subsets. In order to achieve understanding of these notions, (building the notion of quotative division, the writing and reading of this type of division), we should clarify and delimitate the meaning of the expressions: in equal parts, in groups of ... objects, grouped, quotative.

Oral division consists, first of all, of the division of a number formed of integer hundreds to one figure, then to a number formed of hundreds and tens to one figure, each number of tens being exactly dividend to the divisor.

Written division makes the direct and full connection with oral division, because the operation may be easily performed orally as well, a reason for which it is gradually eliminated, as written calculus becomes more advantageous (Lupu, 2011).

2.4. Using set theory and mathematical logic elements in solving problems

The notion of mathematical problem has a wide content and comprises a wide range of preoccupations and actions in different domains. The formative value of solving problems increases, because participation and intellectual involvement in such an activity is higher to other mathematical dimensions, the students having to discover the solving methods and the solution, formulate hypotheses and verify them, make associations of ideas and authentic correlations.

Organizing the problem solving activity relies on the five main stages and mental effort covered by the students, namely: - knowledge of the problem statement; - understanding the problem statement; - problem analysis and schematization; - the solving of the problem proper; verifying the solving of the problem and arranging the solving in the form of an exercise, formulating other problems which are solved according to the same exercise, generalization etc.

Solving simple problems is one of the first steps focused on practicing flexibility and fluency of thought. By solving problems, students eventually operate in a real way, not with numbers, perform operations of composition and decomposition, use anticipative mental strategies and models.

Examining a composed problem is usually done through the *analytical* or *synthetic* method. The two methods may be used simultaneously or, one of them may prevail, a case in which the predominating method imposes its particularities upon the ways leading to finding the solution. Both methods consist in decomposing the given problem into simple problems which, through successive solving, lead to finding the final solution. Practically, the difference between them lies in the starting point of the reasoning. In the *synthesis* method, the starting point is represented by the problem's data used to find its solution, whereas in the *analysis* method the starting point is the problem's question, which leads to its data and establishing the mathematical relations between them. Solving problems according to a solving plan often requires the use of schemes, drawings, graphs etc. and numerical or literal formulas in order to build synthetic thought.

3. Research methodology

3.1. Research hypothesis

Our paper is based on the following research hypothesis: *The use and integration of set theory and mathematical logic elements in lessons of Mathematics will enhance efficient learning of mathematical notions and, by this, the school progress of primary school students.*

In order to demonstrate this hypothesis, we have proposed a psycho-pedagogical research based on a series of research methods: experiment, observation, knowledge testing.

The research hypothesis generates two research variables: - *independent variable* – using and integrating set theory and mathematical logic elements in lessons of Mathematics; - *dependant variable* – increasing the efficiency of acquiring notions, arithmetic operations and, implicitly, the students' school progress. (Dumitriu, 2004).

3.2. Objectives

The essential aim of mathematical education is not reduced to the informative aspect, but teaching this discipline contributes, above all, to developing reasoning and the spirit of receptivity, logical thinking skills, clear and accurate definition of the notions of creative adjustment to current demands.

In order to organize and conduct our practical investigation, we have established the following objectives:

- psychological knowledge of the students, their needs, interests, skills, level of aspirations;

- finding those teaching strategies that aim at activating students during classes of Mathematics;

- discovering the complex stimuli which contribute to educating the students' attention;

- elaborating and applying evaluation tests for the discipline of Mathematics;

- recording, analysing, processing and interpreting the results obtained with a view to establishing the school progress or regress following the application of the didactic game in solving the proposed exercise and problems;

- designing and conducting a methodological process for the discipline of Mathematics by the efficient integration of forms, methods, evaluation techniques characteristic of primary education;

- adopting measures to improve the teaching process, evaluating the results obtained and the students' progress at the end of the year; - elaborating the conclusions of our study.

3.3. Research group

In order to achieve the research objectives and verify the formulated hypothesis, we have included in our research a number of 15 students – 8 girls and 6 boys – in the 4^{th} grade from Scorţeni Middle School, Grigoreni.

The independent variable – the use of set theory and mathematical logic elements in primary education, in solving mathematical exercises and problems.

The dependent variable is the learning performance reflected in the results obtained in the summative evaluation tests, applied to the experimental, respectively control group.

A particularity of this experimental group was the fact that we have acted upon it by means of the experimental factor, the progress factor, in agreement with the research hypothesis, in order to generate change in conducting the educational activity.

Class composition was relatively homogeneous, there being a difference in terms of the girls-boys ration, most of the children having a physical and intellectual normal level of development, 5 students having very good working conditions, 4 having good working conditions, and 6 having satisfactory working conditions. Regarding the parents, 7 have average studies, 5 men work, the mothers being housewives, and 1 student is left to his grandparents' care. There have been certain problems related to the lack of motivation of students belonging to disinterested families.

3.4. Research method

In our research, we have used the following knowledge methods and techniques (Dumitriu, 2004):

- *The method of observation* is frequently used in school, because both spontaneous (passive) observation and scientific (provoked) observation enable the accumulation of rich factual material, being able to provide data on the students' behaviour during lessons, breaks, extracurricular activities and in the family. The teacher's interest is to observe the students' behaviour during certain assigned tasks, how they work, skill, thoroughness, perseverance, initiative, creativity.
- *The method of conversation* has been used to obtain information from students, as well as from parents, the general practitioner or other acquaintances. Thus, we have found interesting aspects related to the children's interests and aspirations, character particularities and skills, general intelligence, family climate, material conditions, the students' daily regime, their health, hobbies. At the same time, we have collected data on the reasons for which they had prepared/ had not prepared for their lessons, their preference/ repulsion for certain activities, possibilities for doing their homework.
- *The analysis of the results/ products of the activity* provide information on aspects which are more difficult to seize through other methods. By product of activity, we understand any material product which is the finalization of an activity conducted by children. The data collected through this method was analyzed by drawing appreciations and estimations regarding the children's individuality, behavior, talents and interests.
- *The biographic method* provides a series of data regarding the students' psychological evolution, in interdependence with the influence of the external factors of development. The data was collected during discussions with parents. Many of the parents of children who have regularly attended kindergarten have argued that their children had made progress particularly in terms of character features, evolving from an internalized temper to a more balanced or even externalized one.
- *The method of the tests* supports the diagnosis of the subject's level of development – in our case, students – and formulating a prognosis of their evolution on this basis. To measure the students' intellectual development, several tests should be applied and the results obtained should be correlated with the results of the other methods applied. *Docimological tests* give quantitative information on the investigated phenomenon. Applied regularly in the

instructive-educational process during lessons of Mathematics and also at other disciplines, they have contributed to determining the level of knowledge, skills and development of intellectual abilities. They were designed in correlation with the established operational objectives, comprising sets of items, which aimed at recording and evaluating school performances and the obtained results were processed and systematized in centralized tables, graphs, histograms, circular diagrams, supporting the data interpretation.

- *The psycho-pedagogical experiment* was the main method used and it was applied to a single class of students, with "*before*" and "*after*" testing. According to the literature, the experiment falls into several categories: the *laboratory experiment* – removing the subject from his usual environment and introducing him into a certain created environment – in our case, the tutored students; the *natural experiment* – the subjects are studied and monitored in usual life situations – in our case, the students were observed in the classroom.
- *Mathematical-statistical methods*: Analytical tables; Synthetic tables; Graphic representations: circular diagram, frequency polygon, histogram; Statistical indices: the mean, the median, the module; Variability indices: amplitude, deviation from the mean or the mean of absolute deviations, dispersion or variant, deviations from the standard or type, the method of the ranks.

3.5. Procedure

The research was conducted during the 2013-2014 school year, on samples of small-age students (8-9 years of age), more precisely during October 20th 2013 – April 30th 2014, at the 4th grade from Scorțeni Middle School, Grigoreni – *experimental group* (progress sample) – represented by 14 students – respectively 8 boys and 6 girls.

The pedagogic experiment implies creating new situations by introducing modifications in the conduct of the educational action, with the aim of verifying the hypothesis, which generated these innovations. Observation may be used in the preliminary phase, as well as while conducting the experiment.

The evaluation tests will be used to measure the volume of knowledge before, during and after the experiment. The basic method will be the psycho-pedagogical, experimental, observation-ameliorative experiment, as well as the method of the tests: initial, formative and final.

The research covered three stages:

- The observation stage, conducted between September 13th, 2013 and October 1st, 2013, consisted in using several methods and procedures with the purpose of knowing the psychic particularities of the students from the 4th grade, from Scorţeni School, Grigoreni. For the discipline of Mathematics, we have applied an initial evaluation test to identify the students' level of knowledge, skills and abilities to apply the acquisitions from previous years. The processing and analysis of these results provided the possibility of formulating conclusions regarding the group of students, as well as each student.
- The formative-ameliorative stage, conducted between October 4th, 2013 and May 25th, 2014, consisted of designing, organizing and applying formative evaluation, through the methods of evaluation for the discipline of Mathematics, as well as the other educational disciplines, with the purpose of engaging all the students in the process of their own training. Based on the obtained results, we have adopted proper measures for organizing differentiated activities also with the students displaying knowledge gaps.
- The final-evaluative stage was conducted at the end of the 2nd semester and consisted of the application of evaluation tests, with the purpose of establishing the students' level of training and their evolution, compared to the initial tests.

4. Results

4.1. Results of initial evaluation

Following the analysis of the results obtained by the students in the initial test, we have found the following:

- the items of exercises 1, 2 were related to the correct solving of the addition and subtraction operations, according to the presence or absence of parentheses;

- we have also looked at how students use mathematical language;

- the difficulty at the first two items derived from the inability to synthesize the information, as students did not manage to write the solving of the problem into an exercise, an aspect also found at the third item;

- analyzing the results, at item 3, only 5 students managed to seize the two ways of solving the problem, which made us focus on stimulating the students' divergent thought and make them look for other solutions to the same problem;

- at item 4, we have obtained, comparatively, 4 Very Good marks and 3 marks of Insufficient, which reveal the fact that at this moment of the research, there are students who cannot compose a problem based on a mathematical expression; this aspect made us insist on building problems by analogy;

This information supported the design of the following activity, given the particularities of each individual student. We have recorded the results obtained by the students in the predictive test in analytical and synthetic centralizing tables. We have graphically represented the data from the centralizing tables in a histogram and circular diagrams.

4.2. Results of formative evaluation

The results of the experimental tests matched our expectations, because the exercises and problems contained 1st order operations, in compliance with the school syllabus for the 2nd grade. The students encountered certain difficulties, especially in relation to the additions and subtractions cu order crossing, the type of problems was identified by all the students, most of them explained accurately and briefly the meaning of the calculi used in the problem solving plan. Analysing the results centralized in tables, we may observe a growth in the number of the students who completed the tests with the Very Good mark, from 5 to 7 students in the end.

4.3. Results of final evaluation

The results obtained in the final test were also represented in the form of histograms and circular diagrams, in order to highlight the progress recorded by the students during the experiment. We shall further present only the comparative analysis of the marks obtained in the initial and final test (Table 1).

Table 1. The results of the initial and final tests

Marks	**Initial stage**	**Final stage**
Sufficient	3 students	2 students
Good	7 students	6 students
Very good	5 students	7 students

When compared, the predictive and final test results have shown the fact that throughout the school year, by the systematic application of active methods and differentiated training during lessons, the progress of students was both qualitative and quantitative. This was seen in the ease and pleasure with which the students acquired a high volume of knowledge, which they used in solving problems and problem situations (knowledge acquired particularly through individual effort), their eagerness to work during the whole school year.

The final evaluation test was designed in a similar way to the initial test, so that the results obtained may be compared, the knowledge established by the syllabus being defined in the form of operational objectives transposed into items.

The analytical and synthetic tables, the diagrams, comparative histogram and frequency polygon highlight the improvement of the students' school results at Mathematics. Synthesizing the results obtained in the two evaluation tests and correlating them with the results obtained in the formative tests, we have found that the students from the 2nd grade have made visible progress regarding the knowledge and use of concepts specific to Mathematics, the ability to solve Arithmetic problems and the ability to communicate using mathematical language.

The students with a lower learning capacity, as a result of being involved in frontal activities, but treated individually, have succeeded in obtaining better marks at the evaluations from the latter part of the school year compared to their results from the beginning, thus becoming more motivated, self-confident and ambitious.

In solving problems, the skills and abilities refer particularly to the data and condition analysis, the capacity to understand the question of the problem and orient the entire development of the reasoning towards discovering the problem solution. The fact that in the final evaluation test 7 students obtained the mark "VG", 6 students – the mark "G" and 2 students – the mark "S", indicates that all the students (100%) have achieved the minimal performances established by the school syllabus for the 2nd grade, 40% - average performances and 47% - maximum performances.

Qualitatively speaking, the work conducted by the students during the experimental period has generated change in their attitude towards learning. The students became more active, more willing to assert and support their ideas, had more initiative and were more self-confident. Because the learning tasks also implied team work, the students displayed better collaboration, higher involvement and more support in achieving the respective tasks.

Conclusions

The use of various applications of set theory and mathematical logic elements in numbering, operations with natural numbers, problem solving, stimulating creative imaginations, involved the use of heuristic strategies.

The tests applied revealed the relevance of the set theory and mathematical logic elements in solving exercises and problems with interest and pleasure. The lessons involving a mathematical didactic game enabled the active participation of students and acquiring knowledge, building an intellectual working style, the lesson turning into a way of organizing the learning activity. Increasing the students' training level through the use of set theory and mathematical logic elements reveals their usefulness for Mathematics, as well as for other disciplines. In Mathematics, the teaching-learning of arithmetic operations with natural numbers and sets has rich formative valences, being an essential way for building the children's independent thinking.

Our paper reveals the inner harmony of Mathematics, its ability to raise awareness of the fact that there are attractive mathematical exercises and problems, whose understanding does not require special talent or training above the level of elementary grades. Our purpose was confirmed and the relevance of set theory and mathematical logic elements for primary education. Identifying them in arithmetic operations was largely due to the students' intellectual skills and the correct acquisition of the various methods for teaching this knowledge.

References

Cerghit, I. (1980). *Metode de învăţământ* [Education methods]. Bucharest: Didactics and Pedagogical Publisher.

Cojocariu, V.-M. (2004). *Teoria şi metodologia instruirii* [Instruction theory and methodology]. Bucharest: Didactics and Pedagogical Publisher.

Cristea, S. (2002). *Dicţionar de pedagogie* [Dictionary of pedagogy]. Chişinău: Litera Educaţional.

Dumitriu, C. (2004). *Introducere în cercetarea psihopedagogică* [Introduction in psycho-pedagogical research]. Bucharest: Didactics and Pedagogical Publisher.

Dumitriu, Gh., Dumitriu, C. (2003). *Psihopedagogie* [Psychopedagogy]. Bucharest: Didactics and Pedagogical Publisher.

Lupu, C. (2011). *Didactica predării aritmeticii* [Didactics of teaching arithmetic]. Bacău: Alma Mater.

Lupu, C. (2014). *Aritmetica pentru învăţători* [Arithmetic for primary teachers]. Bacău: Alma Mater.

Radu, N., Singer, M. (2004). *Matematica, clasa I. Ghid pentru învăţători şi părinţi* [Mathematics, First class. Guide for primary teachers and parents]. Bucharest: Sigma.